Rationale

Mathematics is a tool for everyday life. It is a whole network of concepts and relationships which provide a way of viewing and making sense of the world. It is used to analyse and communicate information and ideas and to tackle a range of practical tasks and real life problems. It also provides the materials and means for creating new imaginative worlds to explore.

Aims and Objectives

- A positive attitude towards mathematics and an awareness of the fascination of mathematics;
- Competence and confidence in mathematical knowledge, concepts and skills;
- An ability to solve problems, to reason, to think logically and to work systematically and accurately;
- Initiative and an ability to work both independently and in cooperation with others;
- An ability to communicate mathematics;
- An ability to use and apply mathematics across the curriculum and in real life;
- An understanding of mathematics through a process of enquiry and experiment.

Organisation and Teaching Approaches

A typical lesson involves all classes following the White Rose scheme of work which is a whole-school primary maths programme. Questions are carefully crafted to develop children's fluency, reasoning, and problem solving skills and conceptual understanding for mastery. It focuses on core topics to build deep understanding.

This policy has been largely adapted from the White Rose Maths Hub Calculation Policy with further material added. It is working document and will be revised and amended as necessary. The purpose of this document is to outline the stages of progress for written calculation methods in the four operations. Each operation uses the concrete, pictorial and abstract (CPA) sequence to support the children in gaining a secure understanding of each concept.

Concrete

In this stage, the children are introduced to an idea or a skill through using real objects. They are able to manipulate the objects and is the foundation for conceptual understanding.

Pictorial
In this stage, the children have understood the hands on experiences performed and can now relate them to representations such as a diagram or picture of the problem.
Abstract
In this stage, the children are now capable of now representing problems by using the mathematical notation. They only use this stage when they have enough context to understand what the calculation means.

It is important that the children are exposed to these three stages in order and are supported by models, images and practical apparatus. These include the use of the part-whole and bar model methods. Examples of these have been included in each section. These should be used regardless of the stage that the children are working at and will need to be returned to when moving to larger numbers.

Teachers need to take into account the stage in which each of their children are working at so that they have a sound understanding of the mathematics and not just a mechanical method for finding an answer. The children need to have this understanding of what they are doing within each stage before moving onto the next. Previous steps therefore may need to be revisited to consolidate understanding when introducing a new strategy.

Number and Place Value
A sound understanding of place value and the number system is essential for children to carry out calculations efficiently and accurately.

Fluency within the four operations is dependent on the children's secure understanding of number and place value. The following skills need to be taken into account;

● Accurate and rapid recall of basic number bonds to 10, 20 and 100.

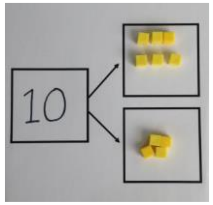
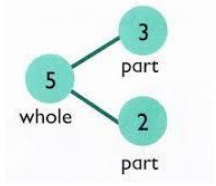
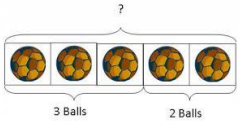
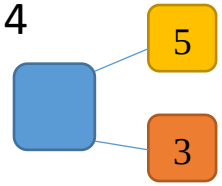
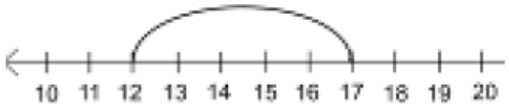
● Having a sense of number by identifying patterns and thinking about connections between calculations.

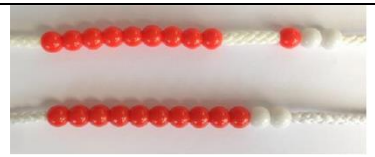
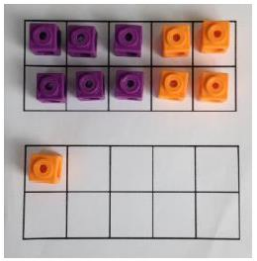
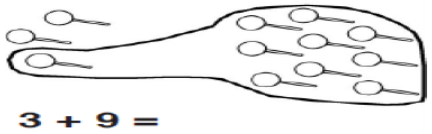
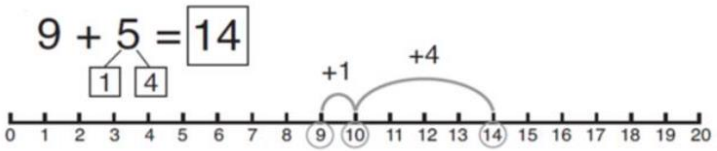
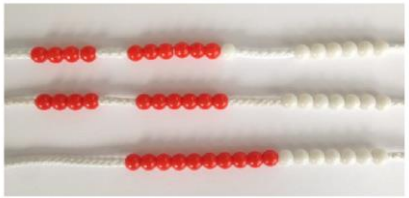
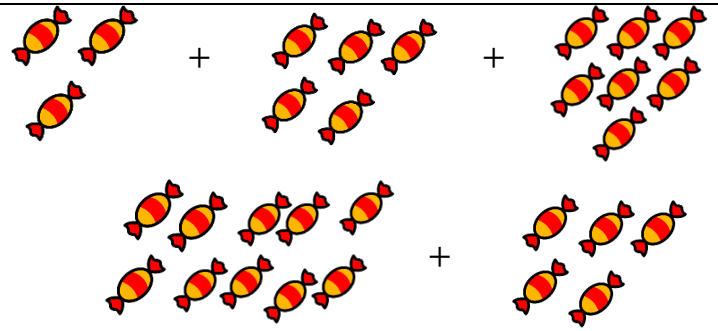
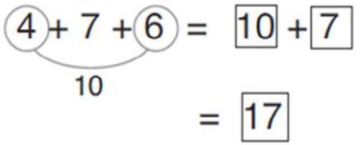
● Accurate and rapid recall of times tables facts (Daily/weekly times tables tests and the use of TT Rockstars). The times tables need to be taught in the following order; x10, x5, x2, x4, x8, x3, x6, x9, x7.

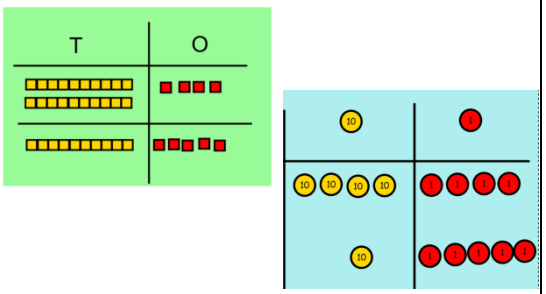
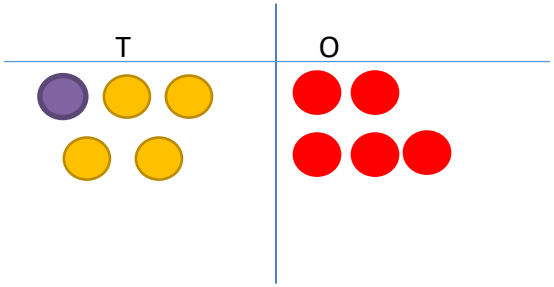
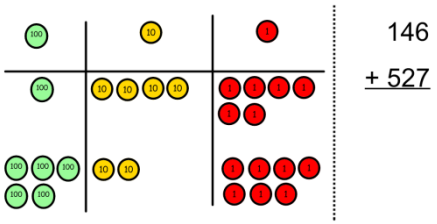
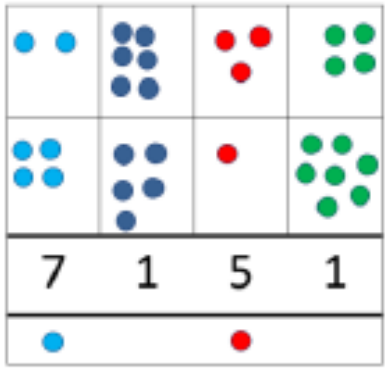
● Developing an understanding of the = symbol (Through the use of balancing scales this concept is easily demonstrated).

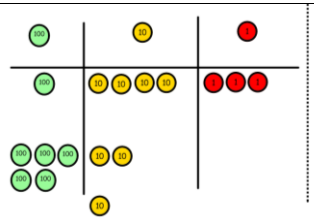
● Developing mathematical reasoning - Through teachers questioning pupils should be encouraged to develop their reasoning skills using the following strategies; What is the same and what is different; odd one out; here's the answer, what could the question have been

Addition

Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	   Use cubes to add two numbers together as a group or in a bar. 	   Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.

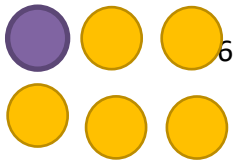
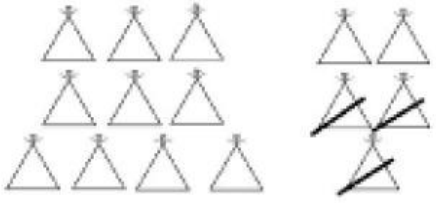
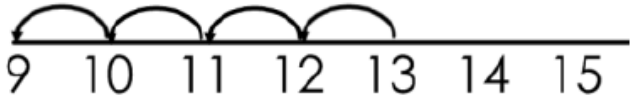
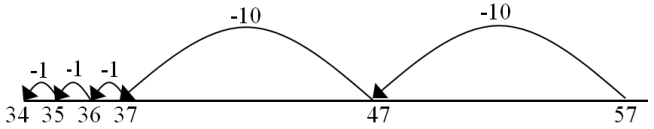
Regrouping to make 10.	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10.	 3 + 9 = Use pictures or a number line. Regroup or partition the smaller number to make 10.  	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?
Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers that make 10 and then add on the remainder.

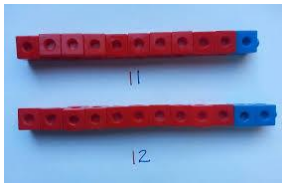
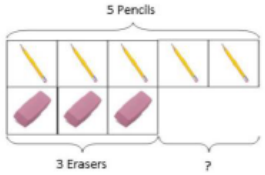
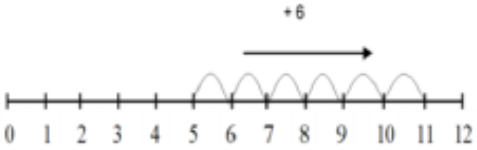
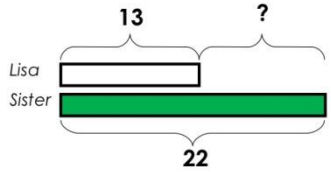
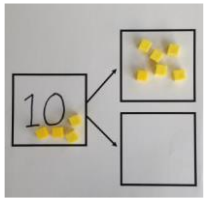
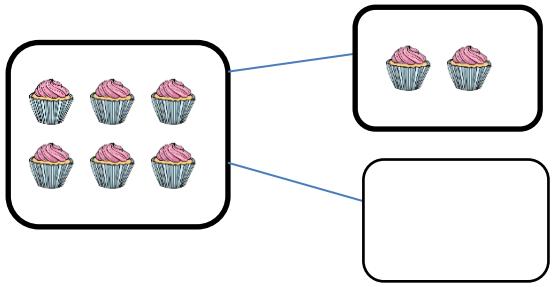
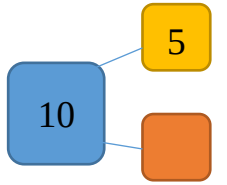
Column method- no regrouping	24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.  	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.  	Calculations 21 + 42 = 21 + 42
Column method- regrouping	Make both numbers on a place value grid.  146 + 527 Add up the units and exchange 10 ones for one 10.	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.  	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. 20 + 5 40 + 8 60 + 13 = 73 536 + 85 621 11

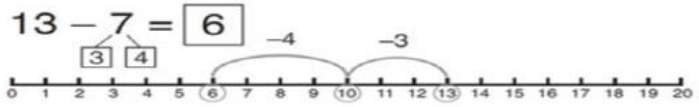
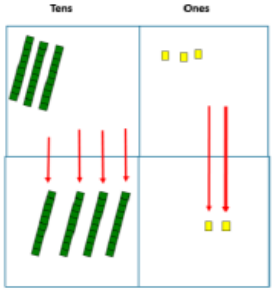
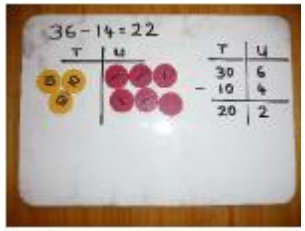
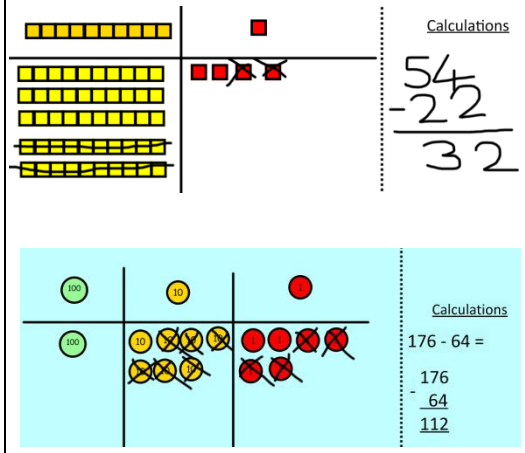
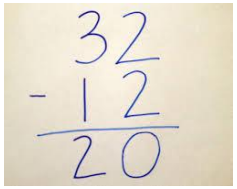
	 146 + 527 Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.		As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here. 72.8 + 54.6 127.4 1 1 £ 2 3 . 5 9 + £ 7 . 5 5 £ 3 1 . 1 4 1 1 1 2 3 . 3 6 1 9 . 0 8 0 5 9 . 7 7 0 + 1 . 3 0 0 9 3 . 5 1 1 2 1 2

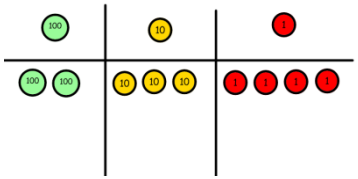
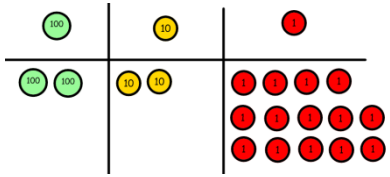
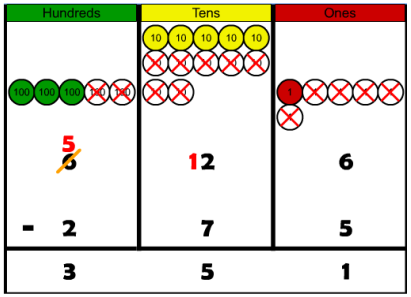
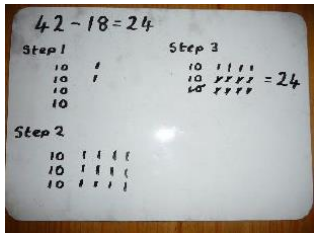
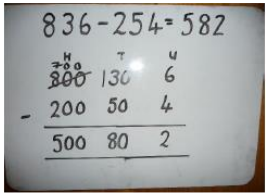
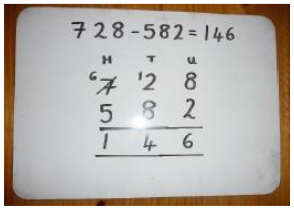
Subtraction

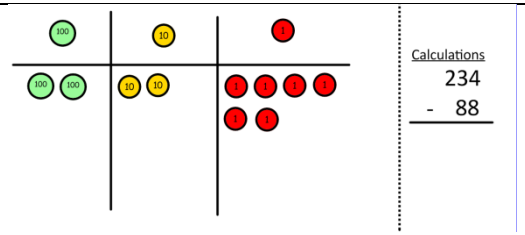
Objective and Strategies	Concrete	Pictorial	Abstract

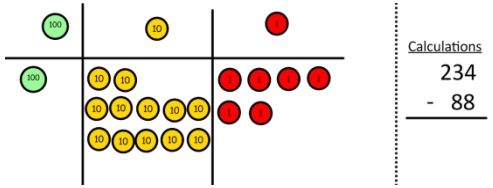
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  6 – 2 = 4 	Cross out drawn objects to show what has been taken away.  15 – 3 = 12	18 -3= 15 8 – 2 = 6
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  13 – 4 Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

Find the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference	 Count on to find the difference. Draw bars to find the difference between 2 numbers. **Comparison Bar Models** *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.*  	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.
Part Part Whole Model	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.
Make 10	14 – 9 =		16 – 8= How many do we take off to reach the next 10?

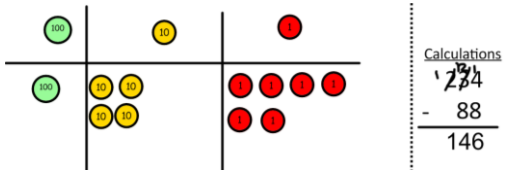
	 Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	How many do we have left to take off?
Column method without regrouping	 Use Base 10 to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first. 	 Draw the Base 10 or place value counters alongside the written calculation to help to show working.	47 − 24 = 23 40 + 7 − 20 + 4 20 + 3 This will lead to a clear written column subtraction. 

Column method with regrouping	Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters  Calculations 234 - 88 Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.  Calculations 234 - 88 Now I can subtract my ones. Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.	 Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.  When confident, children can find their own way to record the exchange/regrouping. Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.	 Children can start their formal written method by partitioning the number into clear place value columns.  Moving forward the children use a more compact method. This will lead to an understanding of subtracting any number including decimals.





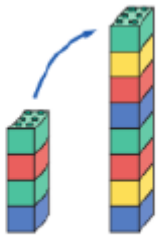
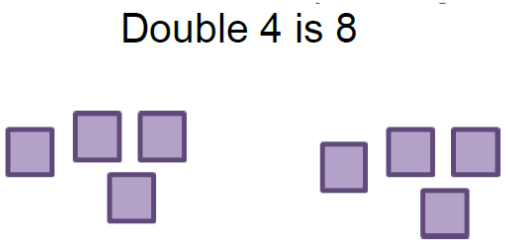
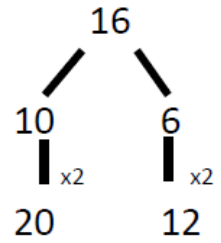
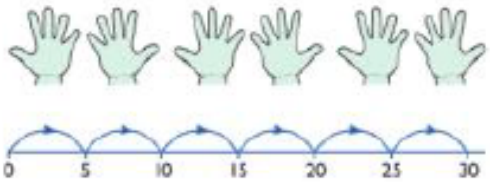
Now I can take away eight tens and complete my subtraction

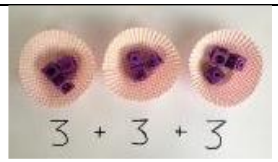
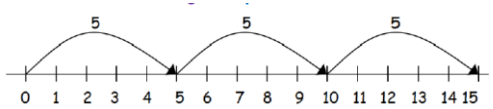
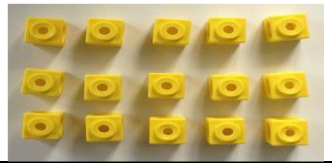
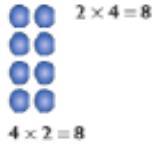
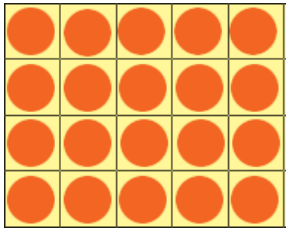


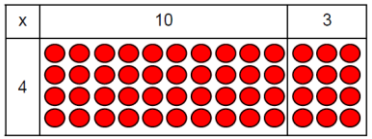
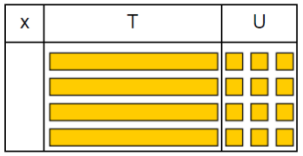
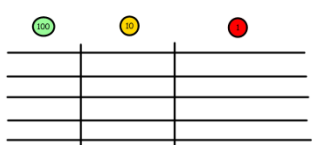
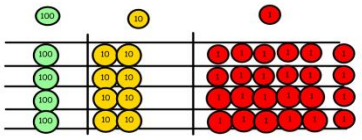
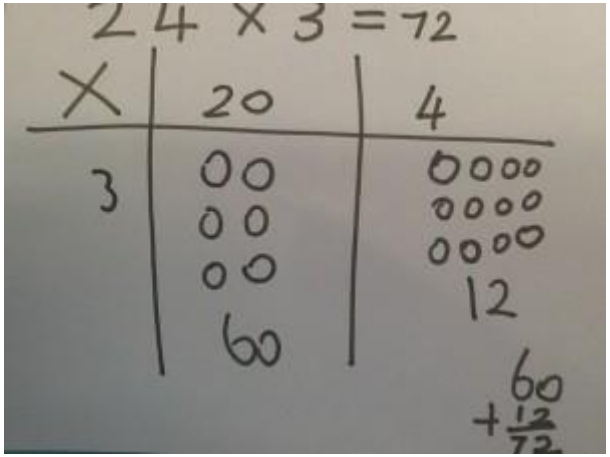
Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

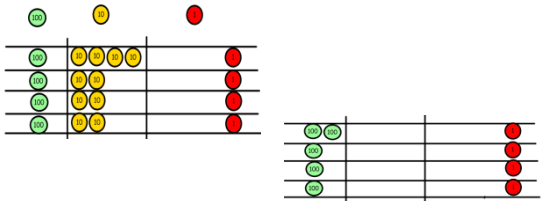
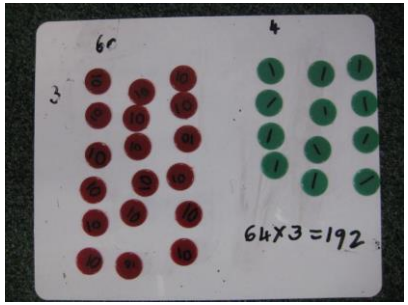
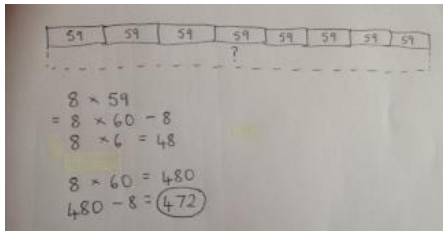
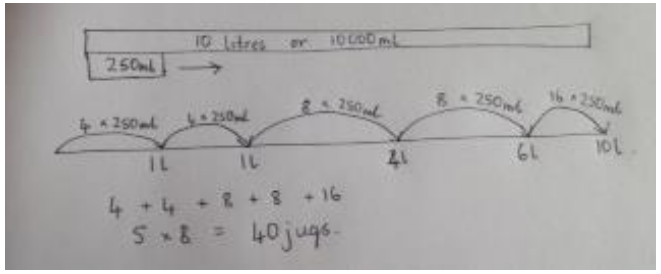
$$\begin{array}{r} {\scriptstyle 5\ \ 12\ \ \ \ 1} \\ 2\ \cancel{6}\ \cancel{3}\ .\ \mathbf{0} \\ -\quad 2\ 6\ .\ 5 \\ \hline 2\ 3\ 6\ .\ 5 \end{array}$$

Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number.  double 4 is 8 $4 \times 2 = 8$	Draw pictures to show how to double a number.  Double 4 is 8	 16 10 x2 6 x2 20 12 Partition a number and then double each part before recombining it back together.
Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 0 5 10 15 20 25 30 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

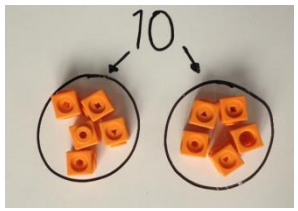
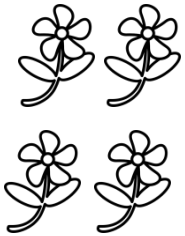
Repeated addition	   Use different objects to add equal groups.	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6  5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures.  2 + 2 + 2 + 2 + 2 = 10
Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.    Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

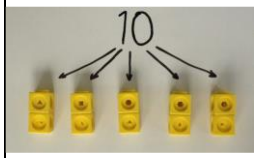
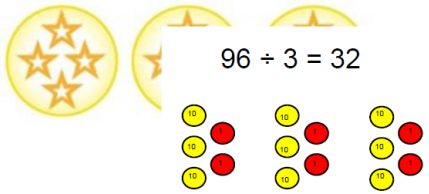
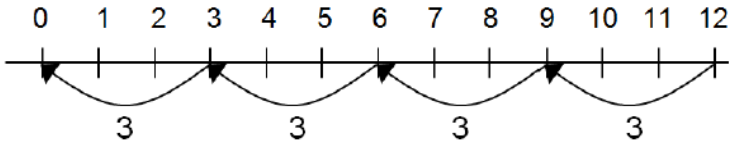
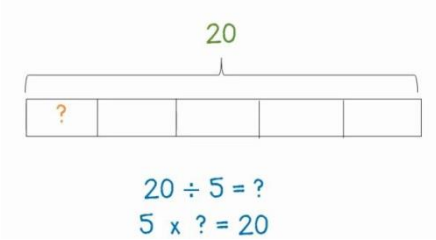
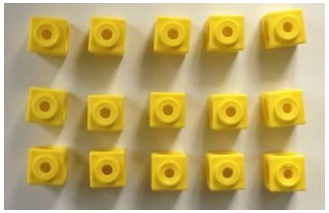
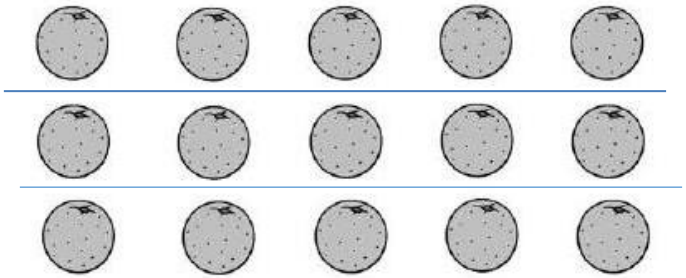
Grid Method	Show the link with arrays to first introduce the grid method.  4 rows of 10 4 rows of 3 Move on to using Base 10 to move towards a more compact method.  4 rows of 13 Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.  Calculations 4 x 126 Fill each row with 126.  Calculations 4 x 126 Add up each column, starting with the ones making any exchanges needed.	Children can represent the work they have done with place value counters in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | × | 30 | 5 | | --- | --- | --- | | 7 | 210 | 35 | **210 + 35 = 245** Moving forward, multiply by a 2 digit number showing the different rows within the grid method. | | 10 | 8 | | --- | --- | --- | | 10 | 100 | 80 | | 3 | 30 | 24 | | X | 1000 | 300 | 40 | 2 | | --- | --- | --- | --- | --- | | 10 | 10000 | 3000 | 400 | 20 | | 8 | 8000 | 2400 | 320 | 16 |

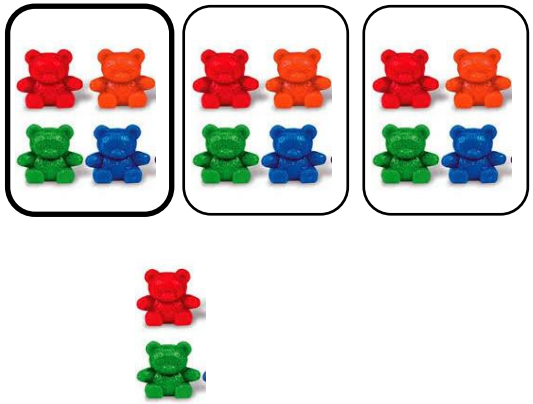
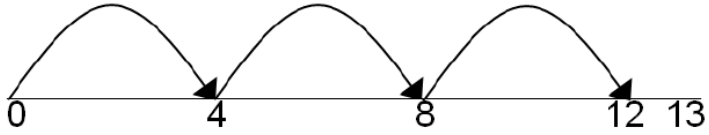
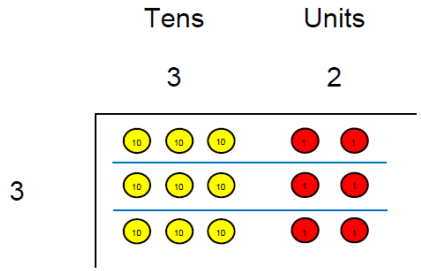
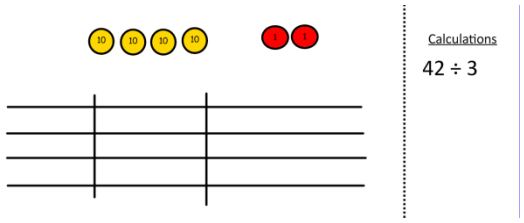
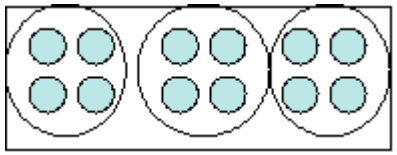
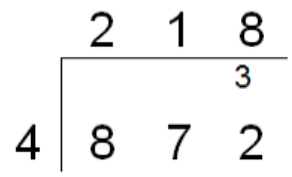
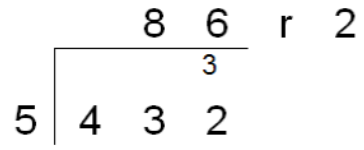
	 Then you have your answer.		
Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication.  It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.  	Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer. 32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768

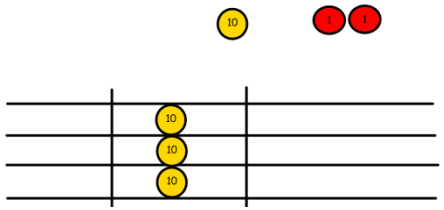
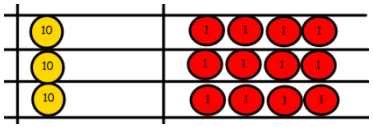
			This moves to the more compact method.  112 X 36 672 3360 4032

Division

Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.   $8 \div 2 = 4$	Share 9 buns between three people. $9 \div 3 = 3$

Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.    	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?
Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7

Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  	Complete written divisions and show the remainder using r. 
Short division	 Use place value counters to divide using the bus stop method alongside  42 ÷ 3=	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder. 

	Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.		Finally move into decimal places to divide the total accurately.